

COMPUTATIONAL STUDY OF THE LAMB WAVE DISPERSION CURVE IN STEEL PLATE

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Abstract- Lamb wave is widely recognized as one of the most promising tools used in solid plates, composite materials and structural characteristics for quantitative identification damage. Lamb waves are regarded as a pioneer in this field of non-destructive testing (NDT). In this study, ANSYS and LS-DYNA software are used to generate the Lamb wave simulation by finite element modeling. Explicit dynamics (LS-DYNA Export) analysis has been used in ANSYS analysis system. Sinusoidal load was applied at the edge of one end of the rectangular plate for the loading case. Lamb wave dispersion curve was calculated in steel plate. The theoretical dispersion curve was derived from the internet source and experimental dispersion curve was attained by numerical simulation. Both theoretical and experimental results are almost identical.

Keywords: Lamb wave, dispersion curves, ANSYS Workbench, LS-DYNA, FEM

1. INTRODUCTION

Lamb waves propagate in solid plates. They are elastic waves whose particle motion lies in the plane containing the propagation direction of the wave and the normal plate. An infinite medium supports only two wave modes roaming at unique speeds; but plates support two infinite Lamb wave modes, whose speeds depend on the connection between wavelength and plate thickness. Surface waves travel to a depth of one wavelength on the surface of a relatively thick solid material. Surface waves combine longitudinal and transverse motion to make an elliptic orbit movement fashionable [1]. Plate waves are parallel to surface waves, except that only a few thick wavelengths can be generated in materials. The most widely encountered plate waves in NDT are Lamb waves. Lamb waves are multi-part vibrational waves that circulate throughout the thickness of the product perpendicular to the sample layer. Lamb wave propagation depends on a component's density and adaptable material properties [2]. They are also influenced by the frequency of the test and the thickness of the material. Lamb waves are formed at an angle of incident where the parallel portion of the wave velocity in the origin is equivalent to the wave velocity in the material of the sample. A variety of types of particle tremor are possible with Lamb waves, although symmetrical and asymmetrical are the two most common. The particles' composite movement is identical to the ground waves' elliptical orbits. Symmetrical Lamb

waves move about the plate's median plane in a symmetrical fashion. This is sometimes referred to as the extensional mode as the wave "stretches and compresses" direction of wave motion [3]. Wave motion in the symmetrical mode is most effectively generated when the exciting force is perpendicular to the surface. The asymmetrical Lamb wave mode is often referred to as the "flexural phase" because a large portion of the motion travels to the plate in a regular direction and a slight displacement happens perpendicular to the plate in the direction. The structure of the plate bends in this manner as the two sides pass in the same direction [4]. The objectives of the study are given below:

1. To Generate Lamb wave in steel plate by numerical simulation.
2. To determine the Lamb wave dispersion curve in steel plate.

1.1 DISPERSION CHARACTERISTICS OF LAMB WAVE

There are three types of bulk waves in an endless (bulk) strong, consisting of longitudinal (P) waves, shear vertical (SV) waves and shear horizontal (SH) waves as shown in Fig. 1. If the solid is limited in isotropic plates by the top and bottom surfaces, the waves of P and SV are merged to produce the waves of Lamb.

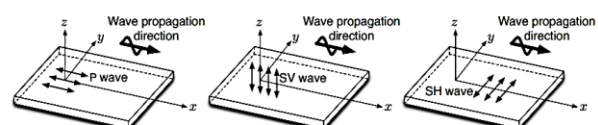
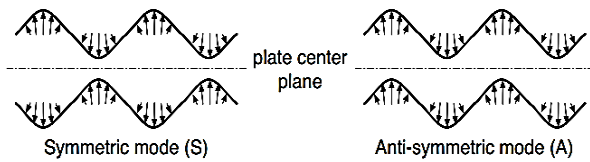


Fig. 1: Bulk waves in solids [5]

The waves of the Lamb propagate in two modes; symmetric (S) and anti-symmetric (A) modes (Fig. 2). The plate displacements in the S modes are symmetrical in relation to their center plane, while in the A modes they are anti-symmetrical. A plate contains an infinite number of modes (S and A). The available modes depend on the excitation frequencies and the thickness of the plate for a specific Lamb wave excitation (Fig. 3). There are more modes at a higher frequency and a larger thickness of the plate. The waves of the Lamb are dispersive because their velocities change with frequencies. There are at least two Lamb modes at each frequency from the dispersion curves of isotropic surfaces. When the excitation frequency increases to a certain frequency limit, the Lamb waves correspond to the Rayleigh waves. It actually implies that the wave frequencies are too high, allowing the waves to propagate only on the plate surface which correlates to



the Rayleigh wave behavior [6].

Fig. 2: The Symmetric and the Anti-symmetric modes [7]

The vibrations of the Lamb are used to challenge the whole thickness of the plates. It is possible to detect both the surfaces or internal damage. However, because of the nature of the multi-modal Lamb waves, only the symmetrical S_0 mode and the anti-symmetrical A_0 mode are normally considered in the literature to improve the analysis of wave signals. There is also some effort to use the shear horizontal SH_0 mode for detection of damage as it is not dispersive in the isotropic plates. Naturally, owing to the anisotropic properties of the product, the Lamb wave propagations in non-homogeneous composite plates are more complicated than in homogeneous isotropic plates. In this scenario, there is a mixture of the P waves, the SV waves and the SH waves. Therefore, it is impossible to solve the Lamb modes and the Love modes independently. The material properties of a typical anisotropic composite plate depend on the properties of the fiber and matrix, the directions of the fiber, the thickness of the lamina and the arrangements in the direction of the plate thickness [8].

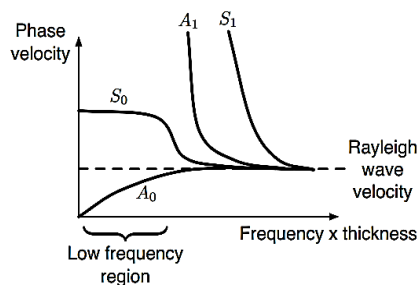


Fig. 3: The typical dispersion curves for the isotropic materials [9]

1.2 LAMB'S CHARACTERISTIC EQUATIONS

For solid materials, elastic waves are directed by the boundaries of the media in which they propagate. Horace Lamb, a spearhead in his day's mathematical physics, established and published the analysis in 1917. The corresponding equations of the Lamb are formalism for a solid plate of infinite length in the x and y directions and thickness d in the z line. Sinusoidal explanations for the wave equation with x and z displacements of the form are postulated –

$$\xi = A_x f_x(z) e^{i(\omega t - kx)}$$

$$\varsigma = A_z f_z(z) e^{i(\omega t - kx)}$$

This form describes the propagation of sinusoidal waves in the x direction with the wavelength of $2\pi/k$ and the amplitude of the wavelength of the spectrum of the wavelength of x. For the free surfaces of the plate, the physical boundary condition is that the component of stress in the z direction at $z=\pm d/2$ is zero. A set of signature equations can be obtained by adding these two assumptions to the above formal solutions to the wave equation. These are:

$$\frac{\tan h(\frac{\beta d}{2})}{\tan h(\frac{\alpha d}{2})} = \frac{4\alpha\beta k^2}{(k^2 + \beta^2)^2}$$

For symmetric modes and

$$\frac{\tan h(\frac{\beta d}{2})}{\tan h(\frac{\alpha d}{2})} = \frac{(k^2 + \beta^2)^2}{4\alpha\beta k^2}$$

For asymmetric modes, where

$$\alpha^2 = k^2 - \frac{\omega^2}{c_l^2} \quad \text{and} \quad \beta^2 = k^2 - \frac{\omega^2}{c_t^2}$$

1.3 LAMB WAVES IN ULTRASONIC TESTING

Generally, the purpose of ultrasonic analysis is to find specific defects in the object being studied and describe them. These faults are observed when the interrupting wave is reflected or dispersed and the wave reflected or dispersed enters the search unit with enough amplitude. Ultrasonic testing has traditionally been performed with waves whose wavelength is much faster than the size of the inspected part. The ultrasonic inspector uses waves in this high-incidence regime that are close to the infinite-medium longitudinal and shear wave methods, zig-zagging across the plate thickness. Acoustic detection requires far lower frequencies than traditional ultrasonic monitoring, and the detector is usually expected to identify active defects up to several meters away. A large section of the structures that are usually tested with acoustic emissions are fabricated from steel plate-tanks, pressure vessels, pipes, etc. Therefore, the Lamb wave hypothesis is the primary concept of understanding the signal forms and velocities of propagation which are found during acoustic emission testing.

1.4 Simulation of Lamb wave

The complexity of the Lamb waves in real world applications is highlighted by many examples in the literature. Numerical simulation is therefore one of the best ways to understand the behaviors of the Lamb wave. In 1967, Viktorov made computations of dispersion curves and plate displacement profiles for isotropic plates using the 90-100 square meter Ural computer. Even composite plates with normal personal and notebook computers (with a 3.4GHz CPU and a 3.5 GB RAM) can now be made in minutes with computer advances. The challenges now are not only to simulate the dispersion curves, but also the propagation of the Lamb wave within the plate itself. From these simulations it is possible to calculate the induced Lamb wave signals from the actuators, the Lamb waves ' interactions with obstacles / damages and the sensor signals. The FEM is the most robust numerical tool available. Such simulations in the FEM, however, still require a lot of computational resources even with current computational power (minimum of ten components per wavelength). These facts have led many researchers to develop other types of numerical methods, i.e. the method of finite difference, the spectral element method, the method of hybrid boundary element (HBE), the method of wave finite element (WFE) and also the method of semi-analytical finite element (SAFE). It is worth noting that the FEM is still widely used for research purposes, especially in complex structures and for comparison and verification purposes, to simulate Lamb wave propagations. The Lamb wave simulation problem in plates can generally be grouped into three main areas; I the calculation of dispersion curves, (ii) the damage / obstacle dispersion of the Lamb wave, and (iii) the combined system simulation of the actuator / sensor [10].

Algor, Abaqus, ANSYS, COSMOS/M, GT-STRUDL, MARC and LS-DYNA software are used for modeling Lamb wave propagation in the plate. Understanding Lamb waves ' interaction with defects, such as delamination, helps to develop effective algorithms for detecting damage. Some preliminary examples are described using Lamb wave scanning techniques to identify internal damage in TOF-based multi-layered composite structures. Ghadami et al [11], for example, proposed an algorithm for identifying parameters of rectangular notch as damage in a plate using Lamb waves. Their algorithm used a combination of pulse-echo and pitch-catch methods. Similarly, shelke et al [12] found that the anti-symmetric mode was more reliable for detection of delamination and changes in Lamb waves ' time-of-flight (TOF) were associated with the progression of damage.

Therefore, for the detection and prediction of delamination interface in composite laminates, a numerical study of the interaction and scattering of Lamb waves by a delamination was first investigated in order to reduce the complexity of Lamb modes in identification. Then, numerically individually derived reflected and transmitted scattering coefficients based on the amplitude and power ratios of the scattered waves. Following a systematic study of the scattering characteristic of A_0 mode at the leading edge of a

delamination, a method based on Lamb waves scattering coefficients was proposed to determine the approximate location of the delamination interface along a laminate thickness [13].

2. METHODOLOGY

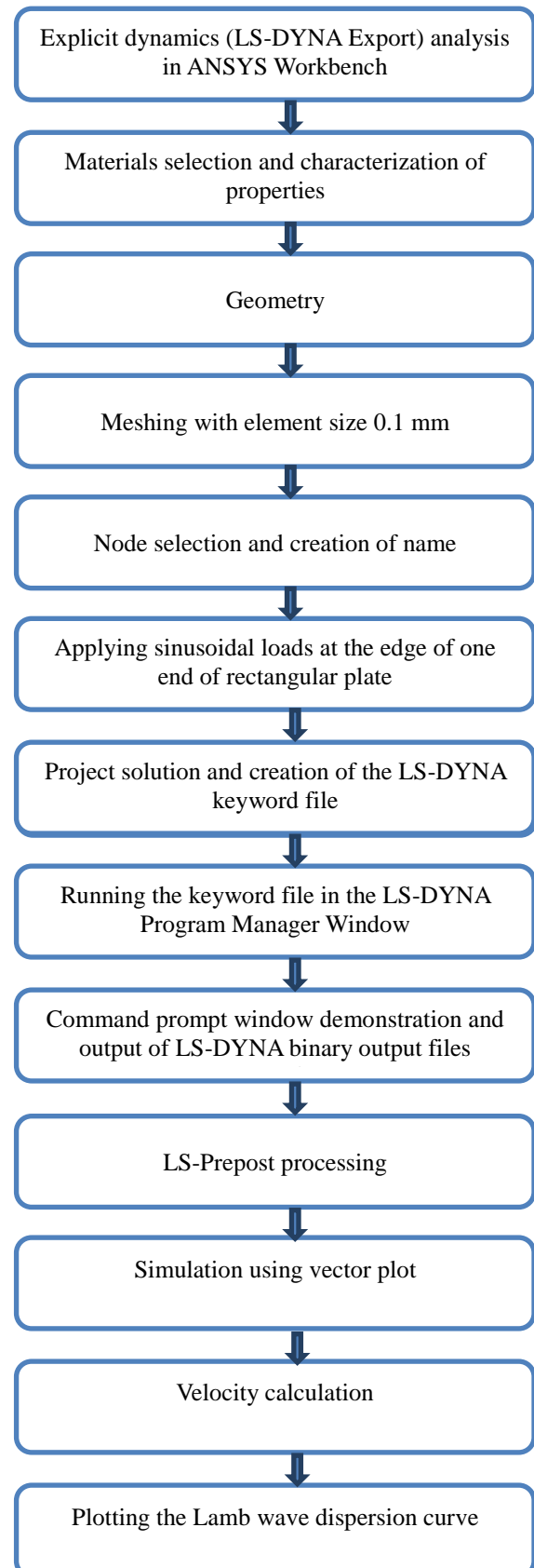


Fig. 4: Flow diagram for the study

(A) ANSYS ANALYSIS SYSTEMS

ANSYS Workbench is a software environment for performing structural, thermal and electromagnetic analysis. Explicit dynamics (LS-DYNA Export) analysis shown in Fig. 5 has been used to perform this study.

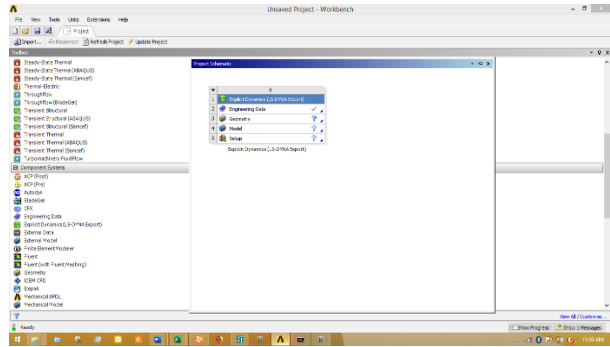


Fig. 5: Explicit Dynamics (LS-DYNA Export) analysis in ANSYS analysis system

(B) ENGINEERING DATA SOURCES

The following figure shows the way to define, process, and organize material properties. Properties can be stored in libraries of materials.

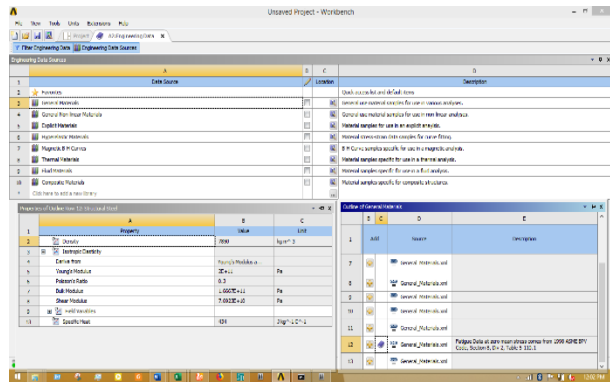


Fig. 6: Engineering data sources

(C) DESIGN MODELER WINDOW

We consider a rectangular plate shown in Fig. 7 into the ANSYS Design Modeler Window and the geometrical dimension of the plate is $75\text{ mm} \times 30\text{ mm}$ for this work.

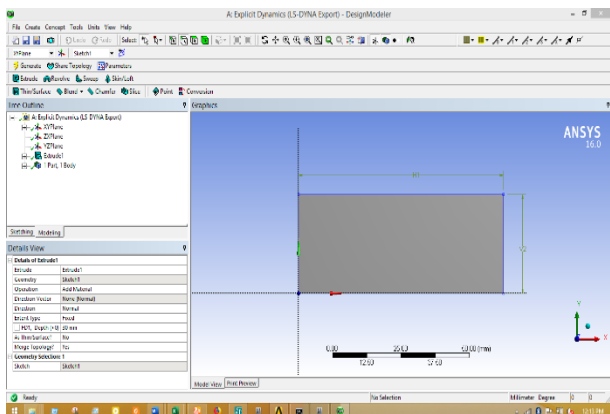


Fig. 7: Drawing rectangle in ANSYS Design Modeler

Window

(D) MESHING

Automatic meshing method has been used in this study to conduct the simulation. In the sizing option of the mesh, the element sizing was set to 0.1 mm and 0.25 mm in thickness and length directions, respectively and the smoothing option was set to high as shown in Fig. 8. Other default options were unaltered and the mesh was generated on each of the designs with the proper enclosure.

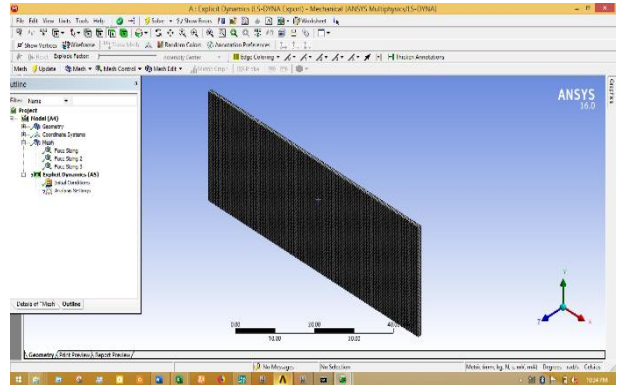


Fig. 8: Meshing the rectangular plate with element size 0.1 mm

(E) NAMED SELECTION

In the Select mode, box volume option has been used for selecting nodes at the edge of one end, as shown in Fig. 9 and then named selection.

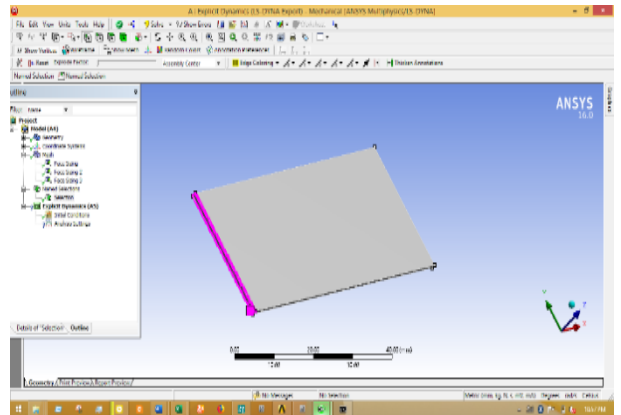
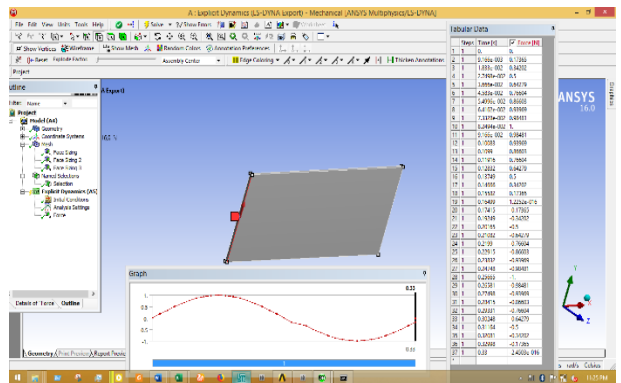


Fig. 9: Selecting nodes at the edge of one end

(F) APPLYING SINUSOIDAL LOADS

For the loading case, sinusoidal load has been applied at the edge of one end of rectangular plate as shown in Fig. 10. The sinusoidal load was added to the rectangular plate with a frequency of 30 kHz within 0.33 sec. Initial time step chosen was 1×10^{-7} sec and time interval between output was taken as 1×10^{-4} sec.



File Tree Solver LS-PrePost M2p... Dyn Variables... Node Linked Controls... License Manager... Main Menu... Help...

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Start Input and Output

Notes: input and output files are mandatory

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Output File:

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ls-dyna [Browse]

Job Path:

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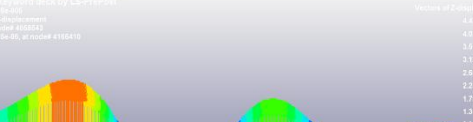
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Fig. 16: Vectors of Z-displacement

For 40 KHz, Vectors of Z-displacement at $t = 3.9898 \times 10^{-5}$ s as shown in Fig. 17.

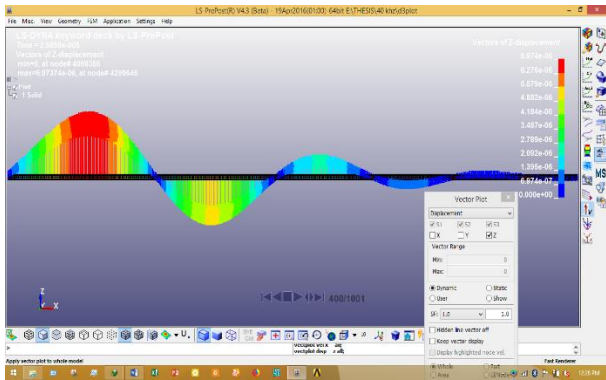


Fig. 17: Vectors of Z-displacement

For 50 KHz, Vectors of Z-displacement at $t = 3.9898 \times 10^{-5}$ s as shown in Fig. 18.

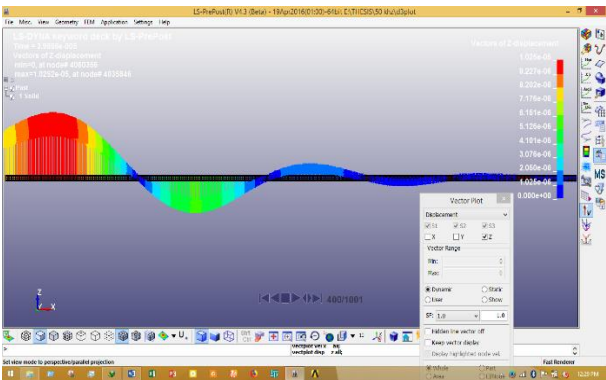


Fig. 18: Vectors of Z-displacement

For 1 MHz, Vectors of Z-displacement at $t = 3.9898 \times 10^{-5}$ s as shown in Fig. 19.

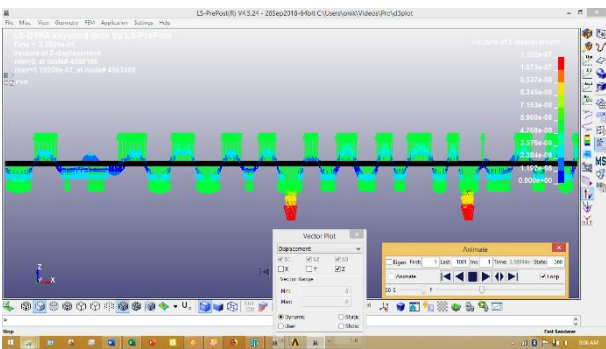


Fig. 19: Vectors of Z-displacement

3.1 CALCULATING VELOCITY

The resultant velocity is calculated according to the following steps:

1. Five points were chosen within the plate at the top surface as shown in Fig. 20.
2. Then resultant velocity was taken from all of them.
3. Then an average of the resultant velocities was calculated.

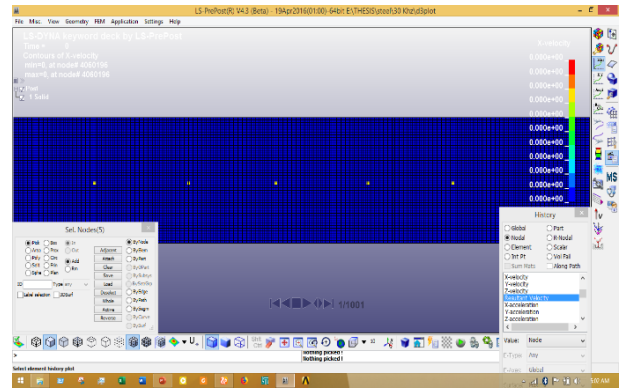
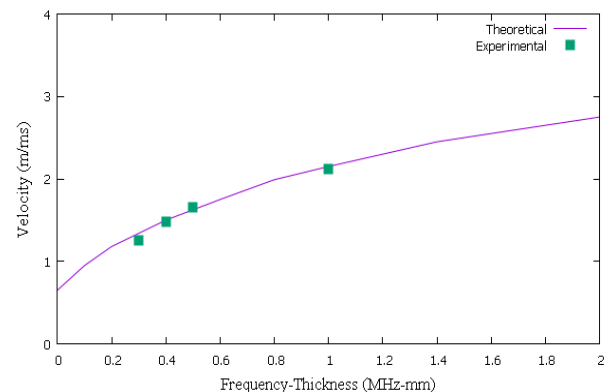


Fig. 20: Selecting nodes in equal distance

3.2 LAMB WAVE DISPERSION CURVE

This demonstrates the relation in a dispersive process between wave velocity, wave length and frequency. The x-axis indicates the angular frequency and plate thickness that is determined by the velocity of the shear wave. The y-axis shows the Lamb wave's phase amplitude, normalized by the velocity of the shear wave. In order to analyze the propagating modes in the controlled material, knowledge of the dispersion curves is essential. There are three methods to plot the dispersion curves: the first is focused on numerical simulations, the second is based on the information obtained after an observation, and the third requires a specific code program like Matlab. For the characteristics (Young modulus, Mass density, Poisson ratio) of this steel plate, theoretical curve is obtained from the Internet source. The experimental curve for the frequency of 30 kHz, 40 kHz, 50 kHz and 1 MHz is used



for numerical simulation as shown in Fig. 21.

Fig. 21: Lamb wave dispersion curve

4. CONCLUSION

Lamb waves have been widely used to detect corrosion, to detect defects in composite materials, in aluminum, in railways, in welded tubes, in roughness solid plates and in the multilayer boards. NDT is used to control defects, corrosion, welding, and other materials. They are used in different positions to detect different types of defects. FEM software offers a wide range of simulation options to manage the complexity of a system's modeling and analysis. FEM helps to produce visualizations of rigidity and strength, as well as minimizing weight, materials and

costs. ANSYS and LS-DYNA software are used in this study to generate the simulation of the Lamb wave through finite element modeling. Lamb wave dispersion curve was calculated in steel plate. The theoretical dispersion curve of this steel plate characteristics was derived from the internet source and experimental dispersion curve was attained by numerical simulation. Both theoretical and experimental results are almost identical. By using this technique, it is possible to find out the scattering characteristics of Lamb wave. If the scattering characteristics are correct then the non-destructive testing technique can be found. Long computation period and intense charging of computer processing units are the major drawback for Lamb wave propagation using 3D finite element method. FEM advantages include improved accuracy, advanced development and greater visibility into crucial project parameters, a more efficient yet shorter design cycle, increased productivity, and increased revenue.

5. REFERENCES

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